If the material is stress free under uniform temperature T_0 at $t = 0$ $[T_0 = T(x,0)]$ with a discrete relaxation spectrum

$$g_{T_0}(\tau) = \sum_{i=1}^{n} G_i(\tau - \tau_i)$$

the strain $e_{ij}(x,t)$ and stress $s_{ij}^{(i)}(x,t)$ associated with the elastic spring G_i are, with the aid of hypothesis I, related by

$$\dot{e}_{ij}(x,t) = \frac{1}{2G_i}\left[s_{ij}^{(i)}(x,t) + \frac{\varphi[T(x,t)]}{\tau_i} s_{ij}^{(i)}(x,t)\right] \quad (14)$$

Equation (14) can be solved for $s_{ij}^{(i)}(x,t)$:

$$s_{ij}^{(i)}(x,t) = 2G_i \int_0^t \dot{e}_{ij}(x,t') \exp\left\{ - \int_{t'}^{t} \varphi[T(x,\lambda)]d\lambda/\tau_i \right\} dt' \quad (15)$$

Hence, with the reduced time ξ of the form of Eq. (6),

$$\begin{aligned} s_{ij}(x,t) &= \sum_{i=1}^{n} s_{ij}^{(i)}(x,t) \\ &= \int_0^t \dot{e}_{ij}(x,t') \sum_{i=1}^{n} 2G_i \exp\{-(\xi - \xi')/\tau_i\}\, dt' \quad (16) \\ &= \int_0^t G_{T_0}(\xi - \xi')\dot{e}_{ij}(x,t')dt' \end{aligned}$$

When the material has a continuous relaxation spectrum $g_{T_0}(\tau)$ initially, the stress-strain relations, with the aid of hypothesis I, are

$$\dot{e}_{ij}(x,t) = \frac{1}{2g_{T_0}(\tau_0)d\tau_0}\left[\frac{\partial s_{ij}(x,t;\tau_0)}{\partial \tau_0}\partial\tau_0 + \frac{\varphi[T(x,t)]\partial s_{ij}(x,t;\tau_0)}{\tau_0 \quad \partial\tau_0} d\tau_0\right] \quad (17)$$

where

$$[\partial s_{ij}(x,t;\ \tau_0)/\partial\tau_0]d\tau_0$$

is the stress associated with the molecules with relaxation times between τ_0 and $\tau_0 + d\tau_0$ at $T = T(x,0) = T_0$ and an invariant spring constant $g_{T_0}(\tau_0)d\tau_0$. From Eq. (17),

$$\frac{\partial s_{ij}(x,t,\tau_0)}{\partial\tau_0} d\tau_0 = 2g_{T_0}(\tau_0)\int_0^t \dot{e}_{ij}(x,t) \exp\left\{-\int_{t'}^{t}\varphi[T(x,\lambda)]d\lambda/\tau_0\right\} dt'$$

Hence, intergrating over τ_0,

$$\begin{aligned} s_{ij}(x,t) &= \int_0^t \dot{e}_{ij}(x,t) \times \int_0^\infty 2g_{T_0}(\tau_0)\exp\{-(\xi - \xi')\tau_0\}\, d\tau_0 d\tau' \quad (18) \\ &= \int_0^t G_{T_0}(\xi - \xi')\dot{e}_{ij}(x,t')dt' \end{aligned}$$

Equations (16) and (18) are identical with Eq. (5) with the reduced time given in Eq. (6). Hence, Eq. (5) with Eq. (6) has been derived directly from hypothesis I.

When the material is viscoelastic in dilatational deformation, the temperature-time equivalence with respect to the bulk relaxation modulus $K(t)$ can be shown to be similar to a hypothesis equivalent to hypothesis I for the relaxation spectrum associated with $K(t)$.

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Inertial Rotation Sensor

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An inertial rotation sensor is a device to measure *absolute* rotations, not merely the rotation relative to an observer. If a man is in a vehicle in space, the rotation sensor fixed to the vehicle should tell him his rate and direction of rotation relative to an inertial frame of reference. In the course of theoretical studies of rotation phenomena it became apparent that various physical phenomena exist which, in principle, could provide mechanisms for inertial rotation-sensing devices, mechanisms that are quite different from those that have been employed in the past. The purpose of this note is to point out one such mechanism.

Suppose a crystal is rotating relative to an inertial frame, and spectroscopic equipment is rotating along with it. The equipment measures the spectral lines of the rotating atoms as seen by an observer who is also rotating. According to a theory given elsewhere,[1] some of the characteristic atomic frequencies seen in such an experiment would differ from the corresponding frequencies in a stationary experiment by an amount proportional to, and of the order of, the rotation frequency itself. Consequently, the frequency shifts could, in principle, be used to detect rotation. In the language of quantum mechanics,[2] the magnitude of the line shift for angular velocity $\boldsymbol{\omega}$ is

$$\Delta\nu_{12} = \left\langle 1\left|\frac{\mathbf{L}\cdot\boldsymbol{\omega}}{h}\right|1\right\rangle - \left\langle 2\left|\frac{\mathbf{L}\cdot\boldsymbol{\omega}}{h}\right|2\right\rangle \quad (1)$$

where $<1|$ and $<2|$ refer to the two states of the atom connected by the line in question, and $\mathbf{L}$ is the internal orbital angular momentum operator for the atom. Since usually the unperturbed energy levels are degenerate, a degeneracy lifted by the rotation, what will actually be seen is a splitting of the line into several components with separation of the order of $\Delta\nu_{12}$. According to (1), the frequency shift occurs if the expectation value of $\mathbf{L}\cdot\boldsymbol{\omega}$ is different in states $<1|$ and $<2|$. For some atoms or ions in crystals these expectation values vanish or are very small, a phenomenon known as orbital quenching. However, in other cases, e.g., the rare earth ions in rare earth salts, these expectation values are of the order of $|\boldsymbol{\omega}|$ for the usual states of the ion.

The experimental problem anticipated is that the line splitting or shift[1] must be observed for a line that has a width much larger than $\Delta\nu_{12}$. Recent development of Masers (microwave range of frequencies) and Lasers (optical range) makes it possible to see changes in the structure or shifts in the center of mass of spectral lines of less than a cycle per second. However, the resolution depends on the shape and width of the line. So favorable conditions for observing the effect are: 1) the two states in question have very different expectation values for $\mathbf{L}\cdot\boldsymbol{\omega}$, and 2) the line is as narrow as possible. One can choose either a line in the paramagnetic resonance spectrum or in the optical spectrum, whichever gives the highest resolution. Finally, it is noted that the frequency shift[1] can be distinguished from the well-known Zeemann effect due to magnetic fields, because the latter acts on the spin as well as on the orbital angular momentum. Otherwise, the details of the rotation effect (i.e., details of the splitting, dependence on polarization, etc.) closely resemble that of the Zeemann effect.

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Received September 14, 1964. This research was supported by NASA Grant 843-NSG-612.

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